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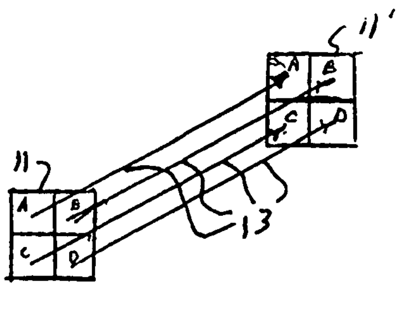

(57) Abstract: In a system for generating a dense motion field representing the motion of image components in a motion picture, a correction to the initial estimate of the dense motion field is directly calculated by determining partial derivatives with respect to time and space from brightness values in two successive frames in the motion picture. The brightness partial derivatives are determined by calculating temporal and spatial differences in brightness values at positions in the successive frames determined by the vectors of the initial estimate of the motion field. The resulting brightness partial derivatives are used to calculate the dense motion field using a motion flow algorithm. The calculated correction to the initial estimate of the dense motion field is then added to the initial estimate to provide a new estimate of the dense motion field. The calculation of the estimated dense motion field is used in the hierarchical pyramid wherein the calculations are carried out on successively finer grids.

WO 01/96982 A2

SYSTEM FOR THE ESTIMATION OF OPTICAL FLOW

Field of the Invention

This invention relates to the estimation of dense motion fields in sequences of images, e.g., video images, by a gradient based optical flow computation.

Background of the Invention

A dense motion field, also called a dense motion vector field, is a set of vectors, one for each pixel of a frame from a set of motion picture frames, wherein the vectors represent the frame to frame movement of pixel-sized image components of the objects depicted in the set of sequential motion picture frames. For example, as shown in Fig. 1, if pixels A, B, C and D represent image components of a depicted square object 11 in a first motion picture frame and the square 11' represents where the square 11 has moved in a second motion picture frame, the vectors 13, representing the change of position of the image components A, B, C and D are vectors of a dense motion vector field. The example of Fig. 1 is a special simplified case in which the square 11 moves without changing its size or orientation. As a result, the dense motion field vectors representing the motion of the image components of the square 11 are parallel and are of equal length. Typically, the motion of objects in a motion picture is more complex than that represented in the example shown in Fig. 1 and the dense motion field vectors will often not be parallel and will not be of equal length. It should be noted that the pixel-sized image components are technically not pixels since pixels by definition do not move. The

image components, on the other hand, are components of the objects depicted in the motion picture and these image components change position from frame to frame when the corresponding objects change position from frame to frame to represent motion of these objects in the motion picture. The computation of a dense motion field is called optical flow computation.

Motion estimation by gradient based optical flow computation between two consecutive images of a sequence has the lack, that the amplitude of motion, which could be determined, is very limited. To overcome this deficiency, the gradient-based method is used as a motion estimation kernel within a hierarchical pyramid as described in PCT application Serial No. WO/9907156, which is hereby incorporated by reference. The skeleton of this pyramid consists of a number of image pairs of decreasing spatial resolution, derived by reducing (down sampling) the original images. At each resolution, a correction to the motion field, determined before at the coarser resolution, is calculated in the motion estimation kernel. After the correction has been added, the refined motion field is filtered and expanded for use in the next finer resolution.

The classical, gradient-based motion estimation kernel algorithm introduced by Horn and Schunck in an article entitled "Determining Optical Flow", published in Artificial Intelligence, 1981, Vol. 17, pp. 185-203, which is hereby incorporated by reference. In the Horn and Schunck approach as described in this article, the image sequence is interpreted as a discrete part of the brightness field $E(t, x, y)$, dense in time t and space (x, y), with the pixels being located at integer positions. The basic assumption of the Horn and Schunck article is that all brightness changes from frame to frame of a motion picture are caused by motion. In other words, each image

component of a depicted object is assumed to stay at the same brightness from frame to frame. This assumption which is called optical flow constraint, can formally be expressed by the vanishing total temporal derivative of the brightness field $E(t, x, y)$, that is:

$$\frac{d}{dt}E=0. \tag{1}$$

Applying the chain rule of differentiation yields the equation

$$E_t+E_x u+E_y v=0, \tag{2}$$

where $u:=\frac{dx}{dt}$ and $v:=\frac{dy}{dt}$

u and v representing the local components of the motion vector (flow velocity), and the indices E_t, E_x and E_y expressing the partial derivatives with respect to time t and space (x, y), that is, $E_t=\frac{\partial}{\partial t}E, E_x=\frac{\partial}{\partial x}E, and\ E_y=\frac{\partial}{\partial y}E$. As it is impossible to get local solutions for both components of the motion vector from a single algebraic equation, Horn and Schunck proposed an additional smoothness constraint for the motion vector field. In order to let the motion vector field approximately fulfill both constraints almost everywhere, they minimize the functional

$$F(u,v):=\int d\,x\,d\,y\,((E_t+E_x u+E_y v)^2+\alpha^2(u_x^2+u_y^2+v_x^2+v_y^2)), \tag{3}$$

wherein the integration extends over the whole space (frame) and the positive constant α^2 controls the relative contributions of the optical flow deviation term and the non-smoothness term. The constant α^2 should be roughly equivalent to the expected noise in the estimate of $E_x^2 + E_y^2$. The minimization is performed using the calculus of variation, such as disclosed in Methods of Mathematical Physics, R. Courant and R. Hilbert, published by Intersciences, New York, New York 1937, 1953. The pertaining

Euler-Lagrange differential equations

$$(E_t + E_x u + E_y v)E_x - \alpha^2(u_{xx} + u_{yy}) = 0 \tag{4}$$

$$(E_t + E_x u + E_y v)E_y - \alpha^2(v_{xx} + v_{yy}) = 0 \tag{5}$$

are re-discretised by replacing the derivatives by discrete difference masks, as explained in the Horn and Schunck article. The resulting system of linear algebraic equations can be solved by standard numerical methods, such as disclosed in <u>Matrix Computation</u>, 2d Ed., by G. H. Golub and C. F. Van Loan, published by Johns Hopkins University Press, Baltimore, MD 1989.

The motion refinement method, used in the above-cited PCT patent application, at each resolution of a hierarchical pyramid, first uses an estimated preliminary motion field (U, V) to warp one of the images called the source image, in order to make a prediction for a second image called the target image. Then, the correction field (u, v) to the preliminary motion field is calculated by estimating the motion (displacement) field between this prediction and the target image. Within the motion estimation kernel of the Horn & Schunck article, partial brightness derivatives E_t, E_x and E_y are calculated as:

$$E_{t,x,y}(t,x,y) := \{ \begin{array}{ll} & E(T^+, X+1, Y+1) \\ (+,-,+) & E(T^+, X\ \ , Y+1) \\ (+,-,-) & E(T^+, X\ \ , Y\ \ ) \\ (+,+,-) & E(T^+, X+1, Y\ \ ) \\ (-,+,+) & E(T, X+1, Y+1) \\ (-,-,+) & E(T, X\ \ , Y+1) \\ (-,-,-) & E(T, X\ \ \ Y\ \ ) \\ (-,+,-) & E(T, X+1, Y\ \ ) \end{array} \}/4. \tag{6}$$

wherein

- the bracketed alternative signs correspond with the equally ordered alternative

variables in the index of the partial derivative (this means that the first symbol, "+" or "-", inside parenthesis is used when determining E_t, the middle symbol inside the parenthesis is used when determining E_x and the last symbol inside the parenthesis is used when determining E_y),

- the brightness values E are calculated at points, expressed with the abbreviations

$$T^{\pm} := t \pm \frac{1}{2}, \quad X := x - \frac{1}{2}, \quad Y := y - \frac{1}{2}. \tag{7}$$

It is emphasized, that the time t lies in the middle between the two images located at consecutive integer times T^- and T^+, that is, $t \in \mathbb{N} + 0.5$. The space points (x, y) lie on the lattice $(\mathbb{N} + 0.5)^2$. The space points (x, y) lie on the lattice $(\mathbb{N} + 0.5)^2$, therefore, X and Y are pixel positions.

Summary of the Invention

As explained above, in the system described in the above-cited PCT application, initial estimates of the motion field are used to predict a target image and then the motion field between the actual target image and the predicted target image is calculated as a correction field. The correction field will be a dense motion field between the predicted target image and the actual target image. This correction field will then be added by vector addition to the initial estimated dense motion field to obtain a new estimate of the dense motion field. In accordance with the present invention, instead of calculating a predicted target image and then determining the correction field from the predicted target image and the actual target image, the

system of the invention calculates the correction motion field directly. In this calculation, the partial derivatives E_t, E_x and E_y are determined from the brightness values in the two successive frames for which the dense motion field is being calculated. More specifically, the brightness partial derivatives are determined by calculating temporal and spatial differences in brightness values at positions in the successive frames determined by the vectors of the initial estimate of the motion field. The resulting brightness partial derivatives are then used to calculate the correction to the dense motion field by means of the algorithm set forth in the Horn and Schunck article.

Brief Description of the Drawings

Fig. 1 is a diagram used to explain a dense motion field which is calculated by the present invention.

Fig. 2 is a block diagram illustrating the system of the present invention.

Fig. 3 is a flow chart of the method of the present invention.

Figs. 4A, 4B and 4C illustrate graphically an example of the coordinate points at which mean brightness values are calculated in accordance with the invention.

Fig. 5 graphically illustrates how the mean brightness value is calculated at a given coordinate point.

Fig. 6 graphically illustrates how an estimate of a Laplacian is calculated.

Fig. 7 is a schematic diagram illustrating the pyramid grid calculating used in the system of the present invention.

Description of a Preferred Embodiment

In the system of the invention, as shown in Fig. 2, a source of successive pixel based motion picture frames are fed to pixel frame buffer memories 21 and 22 wherein the first frame of the sequence is received in the pixel frame buffer memory 21 and the second frame of the sequence is received in pixel frame buffer memory 22. The data processor 24 computes the dense motion field from the brightness values of the pixels in the buffer memories 20 and 22.

As shown in Fig. 3, in the system of the present invention, the data processing unit computes the dense motion field between successive frames of a motion picture by first computing a set of brightness derivatives determined as a function of changes in brightness with space and time and also in accordance with an initial estimate of the dense motion field U and V. Following the computation of these brightness derivatives, the correction to the initial estimate of the dense motion field is calculated using the Horn and Schunck equations. Following this calculation, the correction to the dense motion field is added to the initial estimate to provide a new estimate of the dense motion field.

In accordance with the invention, the CPU 24 calculates the partial brightness derivatives E_t, E_x and E_y taking the preliminary motion field (U, V) into account. They are calculated as follows:

$$E_{t,x,y}(t,x,y,U,V) := \begin{array}{lll} \{ & \bar{E}\,(T^{+} \cdot X_U^{+} + 1, Y_V^{+} + 1) & \\ (+,-,+) & \bar{E}\,(T^{+}, X_U^{+} \quad, Y_V^{+} + 1) & \\ (+,-,-) & \bar{E}\,(T^{+}, X_U^{+} \quad, Y_V^{+} \quad) & \\ (+,+,-) & \bar{E}\,(T^{+}, X_U^{+} + 1, Y_V^{+} \quad) & \\ (-,+,+) & \bar{E}\,(T^{-}, X_U^{-} + 1, Y_V^{-} + 1) & \\ (-,-,+) & \bar{E}\,(T^{-}, X_U^{-} \quad, Y_V^{-} + 1) & \\ (-,-,-) & \bar{E}\,(T^{-}, X_U^{-} \quad Y_V^{-} \quad) & \\ (-,+,-) & \bar{E}\,(T^{-}, X_U^{-} + 1, \; Y_V^{-} \quad) & \}/4. \end{array} \tag{8}$$

The bracketed alternative signs correspond with the equally ordered alternative variables in the index of the partial derivative in the same manner as in Equation (6). The values $\bar{E}$ are mean brightness values and calculated at coordinate points, expressed with the abbreviations

$$X_U^{\pm} := x - \frac{1}{2} + U(x,y) \bullet (\frac{1}{2}(1 \pm 1) - \lambda), \quad Y_V^{\pm} := y - \frac{1}{2} + V(x,y) \bullet (\frac{1}{2}(1 \pm 1) - \lambda). \tag{9}$$

where the parameter $\lambda \in [0,1]$ fixes the time $t = T^{-} + \lambda\,(T^{+} - T^{-})$, whereby the derivatives are calculated, at an arbitrary point between the two times $T^{-} < T^{+}$ belonging to consecutive original images. The space point (x, y) is a point in the frame corresponding to an image component or vector in the initial estimated field. Normally (x, y) will lie either on the lattice $(\mathbb{N} + 0.5)^2$ or on the lattice $\mathbb{N}^2$.

Thus, in Equation (8), each of the eight mean brightness values $\bar{E}$ are determined for specifically identified points in the first motion picture frame or the second motion picture frame. For example, T^{+} in the parenthetical portion of a brightness value $\bar{E}$ means that the brightness value is determined for a point in the second frame and T^{-} means that the brightness value of $\bar{E}$ is determined for a coordinate point in the first frame. The coordinates of the point at which the

brightness value are determined is indicated by the second and third terms in the parenthetical expression. Thus, $\overline{E}(T^{+}, X_U^{+}+1, Y_V^{+}+1)$ means the brightness value is determined for a point in the second frame at the coordinates $X_U^{+}+1, Y_V^{+}+1$ and $\overline{E}(T^{-}, X_U^{-}, Y_V^{-}+1)$ means the brightness value is determined for a coordinate point in the first frame at the coordinates $X_U^{-}, Y_V^{-}+1$. The coordinates $X_U^{+}, Y_V^{+}, X_U^{-}, Y_V^{-}$ are determined from equations (9) by using the plus (+) sign for the plus or minus indicator (±) to compute X_U^{+} and Y_V^{+} and using the minus (-) sign for the plus or minus indicator (±) to compute X_U^{-} and Y_V^{-}.

The points at which the mean brightness values $\overline{E}$ are calculated in Equation (8) are graphically illustrated in Figs. 4A, 4B an 4C for the vector (U,V) which is positioned to pass through the point (x,y), which divides (u,v) in two parts of relative lengths λ and 1-λ. In these figures, λ is about 0.37. In these figures, the eight points at which the mean brightness values are determined are designated 31 through 38. Fig. 4A represents the calculation of E_t. In this figure, the plus (+) signs are on the points 31-34 to indicate that mean brightness values at these points are added in Equation (8) and the minus (-) signs are on the points 35-38 to indicate that mean brightness values at these points are subtracted. In a manner similar to Fig. 4A, Fig. 4B illustrates the calculation of the partial derivative E_x and Fig. 4C illustrates the calculation of the partial derivative E_y. In Figs. 4A-4C, the coordinates at the initial point of the vector and the points 35-38 are in the first frame of the two sequential frames and the coordinates at the terminal point of the vector and the points 31-34 are in the second of the two motion picture frames. As shown in Figs. 4A-4C, the brightness differentials E_t, E_x and E_y are determined by the differences in the mean brightnesses

at locations in the sequential motion picture frames determined in accordance with the initial estimate vector. E_t is determined by the difference between the mean brightness values between the two frames at the initial and terminal points of the corresponding vector. E_x is determined by adding the brightness values at points 31 and 34 in the second frame and at points 35 and 38 in the first frame and subtracting the mean brightness values at the points 32 and 33 in the second frame and at the points 36 and 37 in the first frame. Thus, E_x is determined by differences in mean brightness values at points incrementally spaced in the X direction at the initial point of the vector in the first frame and at the terminal point of the vector in the second frame. Similarly, E_y is determined by adding the mean brightness values at the points 33 and 34 in the second frame and at the points 37 and 38 is the first frame and by subtracting the mean brightness values at the points 31 and 32 in the second frame and at the points 35 and 36 in the first frame. Thus, E_y is determined by the differences in the mean brightness values at incrementally spaced points in the Y direction at the initial and terminal points of the vector in the first and second frames, respectively. The mean brightness $\overline{E}$ are arbitrary convex combinations of the brightness values of the neighboring pixels and each mean brightness value $\overline{E}$ is an approximation of the brightness at the corresponding coordinate point. An approximation is needed because the initial point and termination point of a vector will not be expected to fall at the centers of pixels. One reasonable definition for the mean brightness $\overline{E}$ is given by

$$\begin{aligned}\bar{E}(T,X,Y) := \{ & (1-(X-[X]))(1-(Y-[Y])) && E(T,[X],[Y]) \\ + & (X-[X])(1-(Y-[Y])) && E(T,[X]+1,[Y]) \\ + & (X-[X])(Y-[Y])) && E(T,[X]+1,[Y]+1) \quad \textbf{(10)} \\ + & (1-(X-[X])(Y-[Y])) && E(T,[X],[Y]+1) \quad \},\end{aligned}$$

wherein the coefficients of the convex-combination can be interpreted as the intersection areas of a pixel sized unit square, centered at (X, Y), with the four unit squares, representing its neighboring pixels. The integer positions of the pixels are expressed with the help of clipping brackets, indicating that $[\alpha]$ is the integral part of α, i.e., the largest integer not exceeding α.

The above calculation of Equation (10) computes the $\bar{E}$ brightness approximation as the weighted average of four pixels neighboring the coordinate point for which the $\bar{E}$ brightness approximation is being computed. Fig. 5 graphically illustrates an example of the computation of Equation (10). As shown in Fig. 5, unit square 41 surrounds the coordinate point at (X,Y) for which $\bar{E}$ is being computed. The unit square 41 overlaps the boundaries of four neighboring pixels 43, 45, 47 and 49. $\bar{E}$ is the weighted average of the brightness of the pixels 43, 45, 47 and 49 with each brightness being weighted in accordance with how much it is overlapped by the unit square 41. In this manner, an approximation of the brightness at the coordinate point (X,Y) is determined.

Following the computation of the brightness partial derivatives E_t, E_x and E_y as described above, the equations of Horn and Schunck as set forth in the above-cited article are used to calculate a dense motion field. Because the brightness partial derivatives are determined as an appropriate function of the initial estimate of the

dense motion field, the Horn and Schunck equations will yield a dense motion field which is a correction to the initial estimate and which, when added to the initial estimate, will provide a new estimate of the dense motion field. As described in Horn and Schunck, the partial brightness derivatives can be related to the dense motion field u and v as follows:

$$E_x^2 u + E_x E_y v = \alpha^2 \nabla^2 u - E_x E_t, \tag{11}$$

$$E_x E_y u + E_y^2 v = \alpha^2 \nabla^2 v - E_y E_t. \tag{12}$$

In these equations, $\nabla^2 u$ and $\nabla^2 v$ are the Laplacians of u and v. As explained by Horn and Schunck, the Laplacians of u and v can be approximated by subtracting the magnitudes of u and v from weighted averages of the surrounding magnitudes of u and v as follows:

$$\nabla^2 u \approx K(\bar{u} - u) \quad and \quad \nabla^2 v \approx K(\bar{v} - v), \tag{13}$$

in which $\bar{u}$ and $\bar{v}$ are the weighted averages of the values of u and v surrounding the pixels at which the Laplacians of u and v are being calculated. The weighted average $\bar{u}$ and $\bar{v}$ at the coordinates x,y can be calculated as follows (time dependence suppressed):

$$\begin{aligned}\bar{u}(x,y) &= \frac{1}{6}\{u(x-1),y) + u(x,y+1) + u(x+1,y) + u(x,y-1)\}\\ &+ \frac{1}{12}\{u(x-1,y-1) + u(x-1,y+1) + u(x+1,y+1) + u(x+1,y-1)\}\end{aligned} \tag{14}$$

$$\begin{aligned}\bar{v}(x,y) &= \frac{1}{6}\{v(x-1),y) + v(x,y+1) + v(x+1,y) + v(x,y-1)\}\\ &+ \frac{1}{12}\{v(x-1,y-1) + v(x-1,y+1) + v(x+1,y+1) + v(x+1,y-1)\}\end{aligned} \tag{15}$$

Fig. 6 illustrates the weighting carried out by the above equations for the values at the coordinates x,y. With the approximations of the Laplacians substituted in Equations (11) and (12) and solving for u and v the following equations result:

$$(\alpha^2+E_x^2+E_y^2)u=+(\alpha^2+E_y^2)\bar{u}-E_xE_y\bar{v}-E_{xE_t}, \tag{16}$$

$$(\alpha^2+E_x^2+E_y^2)v=-E_xE_y\bar{u}+(\alpha^2+E_x^2)\bar{v}-E_yE_t. \tag{17}$$

The above equations provide an expression for u and v at each point in the image. These equations can be solved iteratively as follows:

$$u^{n+1}=\bar{u}^n-E_x[E_x\bar{u}^n+E_y\bar{v}^n+E_t]/(\alpha^2+E_x^2+E_y^2), \tag{18}$$

$$v^{n+1}=\bar{v}^n-E_y[E_x\bar{u}^n+E_y\bar{v}^n+E_t]/(\alpha^2+E_x^2+E_y^2). \tag{19}$$

The calculations represented by the above iterative equations are repeated until they converge to provide a dense motion field u and v for each image component. The calculated motion field (u, v) will be a correction to the initial estimate (U, V) and when added to the initial estimate will provide a new estimate of the dense motion field.

The calculation of the estimated dense motion field is then used in a hierarchical pyramid as shown in Fig. 7. In this pyramid, the finest grid 51 corresponds to the pixel display wherein each square of the grid represents one pixel. The other grids 53 and 55 of the pyramid represent progressively coarser grids representing the same image with larger pixels. In accordance with the invention, an initial estimate of the dense motion field is determined for the coarsest grid 55 in the

pyramid and the above-described method is then used to calculate a new estimate of the dense motion field for this coarsest grid 55. This new estimate of the coarsest grid then becomes the initial estimate for the middle grid 53 and a new motion field is calculated by the method described above for the middle grid 53. This new motion field estimate then becomes the initial estimate for the finest grid 51 and the calculation is repeated for the finest grid to produce an estimate of the dense motion field for the finest grid 51. The number of grids in the pyramid is an example and a greater number of grids can be used if desired.

In the above described systems, the equations for solving for the dense motion field have been expressed in the rectangular coordinate system. It will be apparent that the system is not limited to calculations employing rectangular coordinates and other coordinate systems could be used in the solution, such as, for example, polar coordinates.

The above description is of a preferred embodiment of the invention and modification may be made thereto without departing from the spirit and scope of the invention, as defined in the appended claims.

Claims

1. In a method for generating a dense motion field to represent the motion of image components from frame to frame in a motion picture wherein an initial estimate of said dense motion field is made, a correction dense motion field is calculated and said correction dense motion field is added to said initial estimate of said dense motion field to provide a new estimate of said dense motion field, the improvement wherein said correction dense motion field is determined by estimating partial derivatives of the brightness of the images in said motion picture with respect to time and space by the difference of pixel brightness at positions in said motion picture frames determined in accordance with said initial estimate of said dense motion field, and using said partial derivatives of brightness to calculate said correction dense motion field.

2. A method as recited in claim 1, wherein said dense motion field represents the change in position of image components between sequential frames of said motion picture and wherein said differences in pixel brightness are determined for each vector of said initial estimate in the proximity of the corresponding image component at locations in said sequential frames displaced from each other by the corresponding vector.

3. A method as recited in claim 2, wherein the partial derivatives of brightness with respect to time are determined for each vector of said initial estimate by the differences in brightness of pixels between said sequential frames and the partial derivatives of brightness with respect to space being determined by incremental

differences in the brightnesses between pixels in both of said sequential frames.

4. A method as recited in claim 3, wherein the brightness partial derivatives with respect to time E_t and with respect to space E_x and E_y are calculated as:

$$
\begin{array}{lll}
E_{t,x,y}(t,x,y,U,V) := & \{ & \overline{E}\,(T^{+}, X_{U}^{+}+1, Y_{V}^{+}+1) \\
 & (+,-,+) & \overline{E}\,(T^{+}, X_{U}^{+}\ \ , Y_{V}^{+}+1) \\
 & (+,-,-) & \overline{E}\,(T^{+}, X_{U}^{+}\ \ , Y_{V}^{+}\ \ ) \\
 & (+,+,-) & \overline{E}\,(T^{+}, X_{U}^{+}+1, Y_{V}^{+}\ \ ) \\
 & (-,+,+) & \overline{E}\,(T^{-}, X_{U}^{-}+1, Y_{V}^{-}+1) \\
 & (-,-,+) & \overline{E}\,(T^{-}, X_{U}^{-}\ \ , Y_{V}^{-}+1) \\
 & (-,-,-) & \overline{E}\,(T^{-}, X_{U}^{-}\ \ \ Y_{V}^{-}\ \ ) \\
 & (-,+,-) & \overline{E}\,(T^{-}, X_{U}^{-}+1, Y_{V}^{-}\ \ ) \qquad \}/4.
\end{array}
$$

in which the values E bar [do elsewhere] are mean brightness values estimated at coordinate points and successive frames of said motion picture and wherein

$$X_{U}^{\pm} := x - \frac{1}{2} + U(x,y) \bullet (\frac{1}{2}(1 \pm 1) - \lambda), \ \ Y_{V}^{\pm} := y - \frac{1}{2} + V(x.y) \bullet (\frac{1}{2}(1 \pm 1) - \lambda).$$

in which λ is a value between 0 and 1 and x and y are the coordinate of a coordinate point in the motion picture frames.

5. In a system for generating a dense motion field comprising storage means connected to receive and store successive frames of a motion picture and a data processor connected to receive the data representing said motion picture frames and programmed to generate a dense motion field representing the motion of image components from frame to frame, wherein an initial estimate of said dense motion field is made and wherein said data processor is programmed to calculate a correction dense motion field which is added to the initial estimate of said dense motion field to

provide a new estimate of said dense motion field, the improvement wherein said data processor is programmed to determine said correction dense motion field by estimating partial derivatives of brightness of the images in said motion picture with respect to time and space by the difference of the pixel brightness at positions in said motion picture frames determined in accordance with said initial estimate of said dense motion field and using said partial derivatives of brightness to calculate said correction dense motion field.

6. A system as recited in claim 5, wherein said dense motion field represents a change in position of image components between sequential frames of said dense motion field stored in said storage means and wherein said data processor determines said differences and pixel brightness for each vector of said initial estimate and the proximity of the corresponding image component at locations in said sequential frames displaced from each other by the corresponding vector.

7. A system as recited in claim 6, wherein the partial derivatives of brightness with respect to time are determined for each vector of said initial estimate by the differences in brightness of pixels between said sequential frames and the partial derivatives of brightness with respect to space are determined by incremental differences in the brightnesses between pixels in both of said sequential frames.

8. A system as recited in claim 7,wherein said data processor is programmed to determine the brightness partial derivative with respect to time E_t and the brightness derivatives with respect to space E_x and E_y in accordance with

$$
\begin{array}{lll}
E_{t,x,y}(t,x,y,U,V) := & \{ & \bar{E}\,(T^{+} \cdot X_{U}^{+}+1, Y_{V}^{+}+1) \\
 & (+,-,+) & \bar{E}\,(T^{+}, X_{U}^{+} \quad , Y_{V}^{+}+1) \\
 & (+,-,-) & \bar{E}\,(T^{+}, X_{U}^{+} \quad , Y_{V}^{+} \quad) \\
 & (+,+,-) & \bar{E}\,(T^{+}, X_{U}^{+}+1, Y_{V}^{+} \quad) \\
 & (-,+,+) & \bar{E}\,(T^{-}, X_{U}^{-}+1, Y_{V}^{-}+1) \\
 & (-,-,+) & \bar{E}\,(T^{-}, X_{U}^{-} \quad , Y_{V}^{-}+1) \\
 & (-,-,-) & \bar{E}\,(T^{-}, X_{U}^{-} \quad Y_{V}^{-} \quad) \\
 & (-,+,-) & \bar{E}\,(T^{-}, X_{U}^{-}+1, Y_{V}^{-} \quad) \qquad \}/4.
\end{array}
$$

in which the values E bar [fix on another machine] are mean brightness values calculated at coordinate points in successive frames of said motion picture expressed with the abbreviations

$$X_{U}^{\pm} := x - \frac{1}{2} + U(x,y) \bullet (\frac{1}{2}(1 \pm 1) - \lambda), \quad Y_{V}^{\pm} := y - \frac{1}{2} + V(x.y) \bullet (\frac{1}{2}(1 \pm 1) - \lambda).$$

in which λ is a value between 0 and 1 and x and y are the coordinates of coordinate points in the motion picture frames.

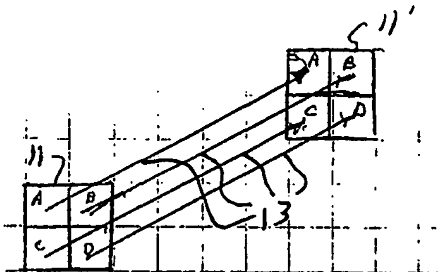
11'
A
B
C
D
11
A
B
C
D
13

Fig 1

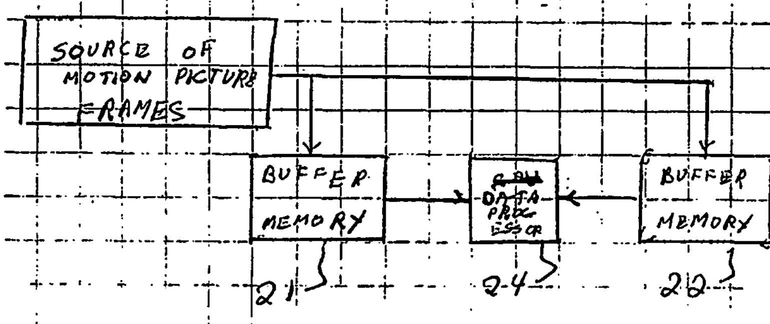
SOURCE OF MOTION PICTURE FRAMES
BUFFER MEMORY
DATA PROCESSOR
BUFFER MEMORY
21
24
22

Fig 2

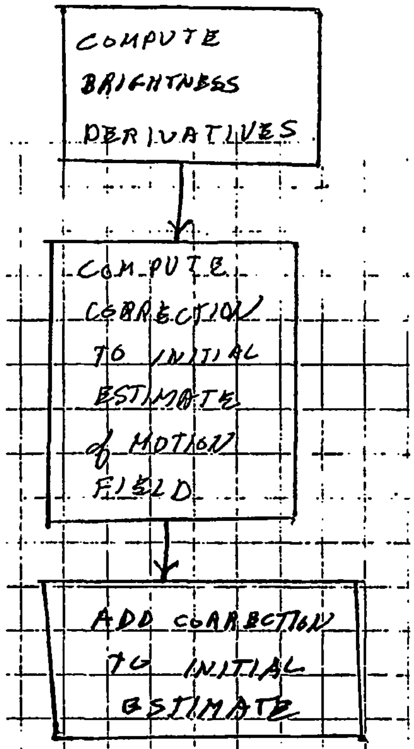
COMPUTE BRIGHTNESS DERIVATIVES
COMPUTE CORRECTION TO INITIAL ESTIMATE of MOTION FIELD
ADD CORRECTION TO INITIAL ESTIMATE

FIG 3

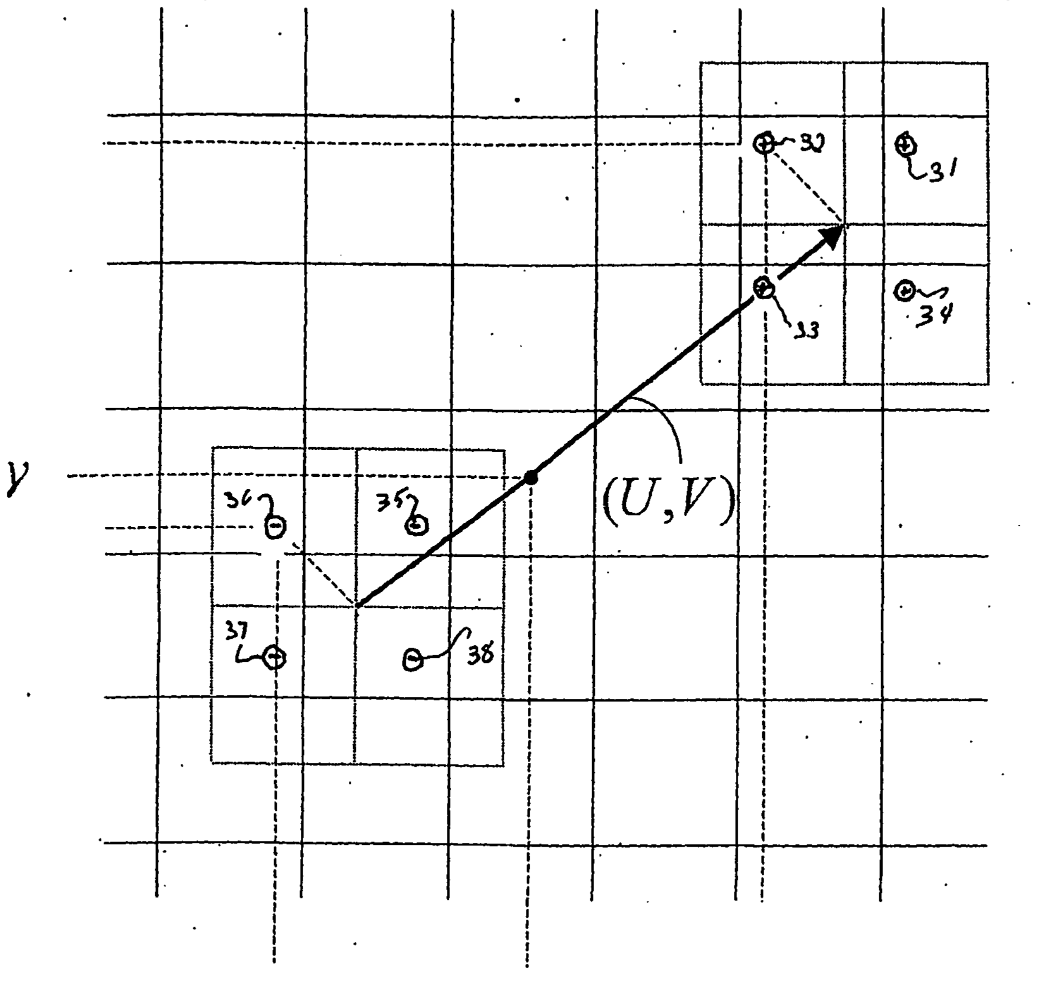

FIG. 4A

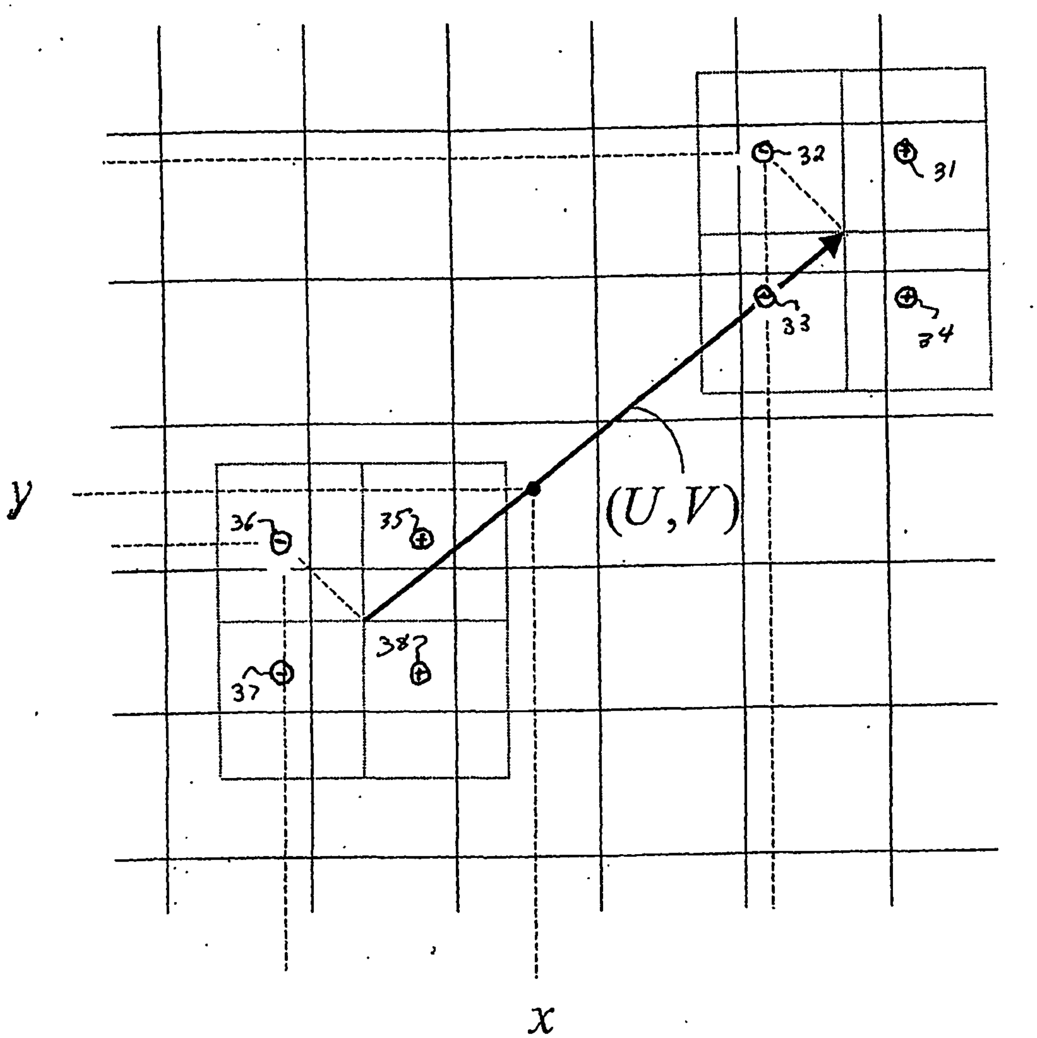

FIG 4B

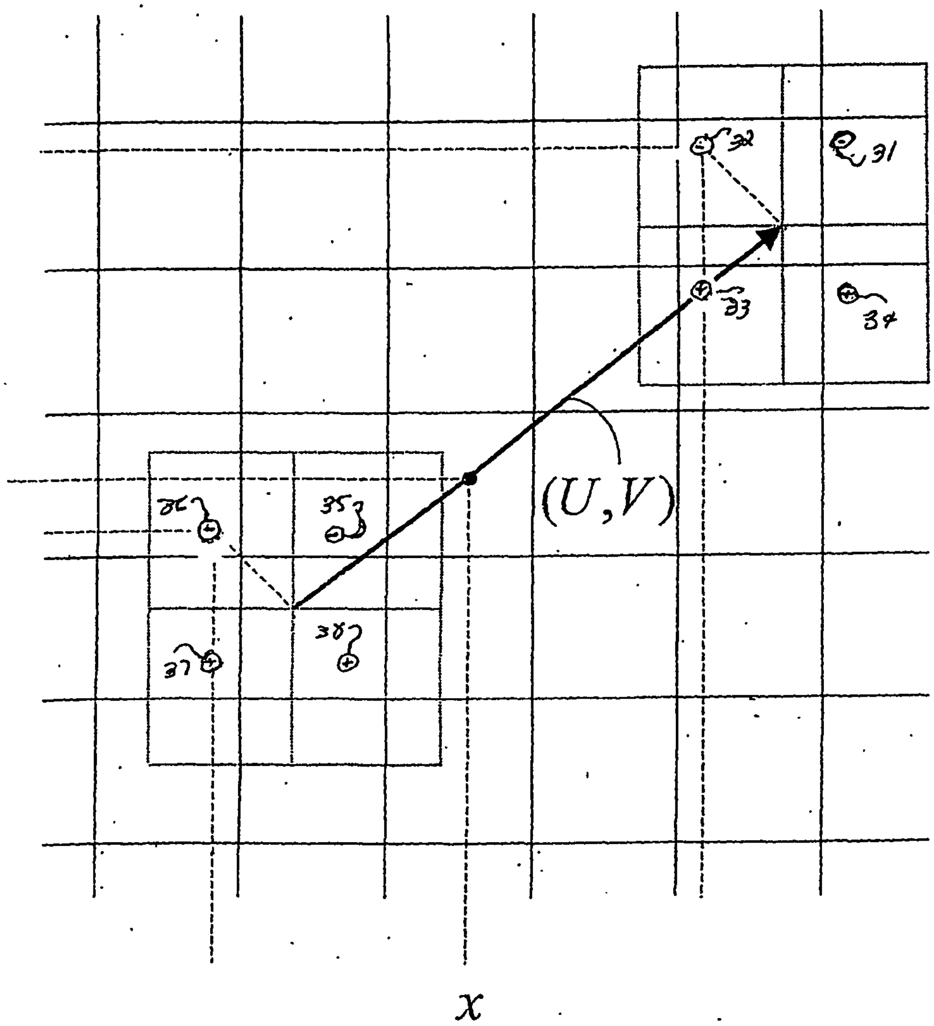

Fig 4C

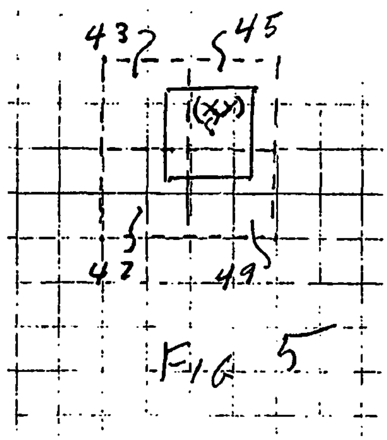

FIG 5

1/12 x-1, y+1	1/6 x, y+1	1/12 x+1, y+1
1/6 x-1, y	x, y	1/6 x+1, y
1/12 x-1, y-1	1/6 x, y-1	1/12 x+1, y-1

FIG 6

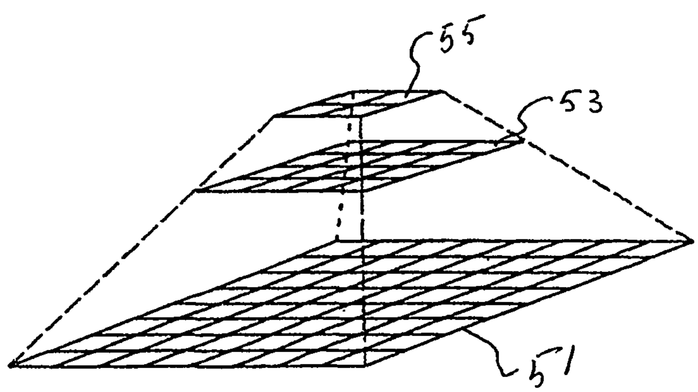

FIG. 7